General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS **MPC4**
Unit Pure Core 4

Thursday 12 June 2008 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The polynomial $f(x)$ is defined by $f(x) = 27x^3 - 9x + 2$.

(a) Find the remainder when $f(x)$ is divided by $3x + 1$. *(2 marks)*

(b) (i) Show that $f\left(-\frac{2}{3}\right) = 0$. *(1 mark)*

(ii) Express $f(x)$ as a product of three linear factors. *(4 marks)*

(iii) Simplify

$$\frac{27x^3 - 9x + 2}{9x^2 + 3x - 2}$$ *(2 marks)*

2 A curve is defined, for $t \neq 0$, by the parametric equations

$$x = 4t + 3, \quad y = \frac{1}{2t} - 1$$

At the point P on the curve, $t = \frac{1}{2}$.

(a) Find the gradient of the curve at the point P. *(4 marks)*

(b) Find an equation of the normal to the curve at the point P. *(3 marks)*

(c) Find a cartesian equation of the curve. *(3 marks)*

3 (a) By writing $\sin 3x$ as $\sin(x + 2x)$, show that $\sin 3x = 3\sin x - 4\sin^3 x$ for all values of x. *(5 marks)*

(b) Hence, or otherwise, find $\int \sin^3 x \, dx$. *(3 marks)*

4 (a) (i) Obtain the binomial expansion of $(1 - x)^{\frac{1}{4}}$ up to and including the term in x^2. *(2 marks)*

(ii) Hence show that $(81 - 16x)^{\frac{1}{4}} \approx 3 - \frac{4}{27}x - \frac{8}{729}x^2$ for small values of x. *(3 marks)*

(b) Use the result from part (a)(ii) to find an approximation for $\sqrt[4]{80}$, giving your answer to seven decimal places. *(2 marks)*

5 (a) The angle α is acute and $\sin\alpha = \frac{4}{5}$.

(i) Find the value of $\cos\alpha$. *(1 mark)*

(ii) Express $\cos(\alpha - \beta)$ in terms of $\sin\beta$ and $\cos\beta$. *(2 marks)*

(iii) Given also that the angle β is acute and $\cos\beta = \frac{5}{13}$, find the exact value of $\cos(\alpha - \beta)$. *(2 marks)*

(b) (i) Given that $\tan 2x = 1$, show that $\tan^2 x + 2\tan x - 1 = 0$. *(2 marks)*

(ii) Hence, given that $\tan 45° = 1$, show that $\tan 22\frac{1}{2}° = \sqrt{2} - 1$. *(3 marks)*

6 (a) Express $\dfrac{2}{x^2 - 1}$ in the form $\dfrac{A}{x-1} + \dfrac{B}{x+1}$. *(3 marks)*

(b) Hence find $\displaystyle\int \frac{2}{x^2 - 1}\, dx$. *(2 marks)*

(c) Solve the differential equation $\dfrac{dy}{dx} = \dfrac{2y}{3(x^2 - 1)}$, given that $y = 1$ when $x = 3$.

Show that the solution can be written as $y^3 = \dfrac{2(x-1)}{x+1}$. *(5 marks)*

7 The coordinates of the points A and B are $(3, -2, 1)$ and $(5, 3, 0)$ respectively.

The line l has equation $\mathbf{r} = \begin{bmatrix} 5 \\ 3 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$.

(a) Find the distance between A and B. *(2 marks)*

(b) Find the acute angle between the lines AB and l. Give your answer to the nearest degree. *(5 marks)*

(c) The points B and C lie on l such that the distance AC is equal to the distance AB. Find the coordinates of C. *(5 marks)*

Turn over for the next question

8 (a) The number of fish in a lake is decreasing. After t years, there are x fish in the lake. The rate of decrease of the number of fish is proportional to the number of fish currently in the lake.

(i) Formulate a differential equation, in the variables x and t and a constant of proportionality k, where $k > 0$, to model the rate at which the number of fish in the lake is decreasing. *(2 marks)*

(ii) At a certain time, there were 20 000 fish in the lake and the rate of decrease was 500 fish per year. Find the value of k. *(2 marks)*

(b) The equation

$$P = 2000 - Ae^{-0.05t}$$

is proposed as a model for the number of fish, P, in another lake, where t is the time in years and A is a positive constant.

On 1 January 2008, a biologist estimated that there were 700 fish in this lake.

(i) Taking 1 January 2008 as $t = 0$, find the value of A. *(1 mark)*

(ii) Hence find the year during which, according to this model, the number of fish in this lake will first exceed 1900. *(4 marks)*

END OF QUESTIONS